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FORWARD

Who Is This Book For?

I wrote this book for anyone and everyone. It is for the ***common person*** as well as the math enthusiast. It is for the students who want to enhance their learning. It is for the ***student*** who is struggling and needs a book that explains things in plain terms with many examples. It is for the ***parent*** who is helping their child at home or who is going back to school. It is for the ***teacher*** who is planning math lessons and needs a quick reference to easily access the topics and prerequisite topics that need to be taught with examples and exercises without spending hours trying to piece it together on the internet or with multiple books that have as a goal trying to complete a checklist that satisfies the latest trend in teaching, learning, and curriculum development, as well as evaluation of schools, students, teachers, and administrators. It is for ***administrators*** and ***people at a decision-making level in the government*** who wish to design curriculum that is less ambiguous and more relevant, so it is clear what must be taught and how it should be taught, with the ultimate goal a highly effective more uniform math curriculum and most importantly, a math curriculum that is ***equally accessible to all***.

It is a great reference for ***students from grade school through college***. I wrote it in plain language rather than theoretical language. It starts with sets and some concepts a young student might not be developmentally ready for with respect to a certain level of complexity and depth of understanding, as it was developed in a very precise and strict manner in which concepts (and sub-concepts) are introduced and added to as they aid in the understanding of each new concept. However, ***it is still a great resource for teachers and parents of students of all ages*** both in planning what needs to be taught and in deciding which methods are going to be more effective if the knowledge gained is going to be used at a higher level. Adjustments may have to be made as to how it is presented and acquired at a younger age such as listing sets of animals and spelling and reciting the ten digits of the base 10 system, and understanding can be further enhanced as each individual learner's developmental stage evolves. The bottom line is the book is in the order one would learn math if when they started reading the book, they were already at a mature developmental stage such as high school age where they possessed both the thinking ability to acquire and the experiences to synthesize the knowledge at a higher level. By writing it this way, the book does not get bogged down in trying to adjust for developmental abilities as, although there ***is*** a lot of overlap and revisiting of concepts, a book that attempts to do this would be an exercise in chaos for the learner. At a higher level, many of my high school and community college students have great gaps in their critical math skills such as prime factorization and its many uses in learning other skills. Their books are many times too condensed, too narrowly focused, at a hurried pace, and without sufficient examples. I can easily pull from my book any prerequisite skills that are needed to understand the concepts with which they are having difficulty.

This is a great book for parents of children in public, private, and alternative schools as well as parents of homeschooled children as they are many times left in the dark as to what their children are learning, and which methods are being used to learn the concepts being taught. Curriculums can vary from state to state, school to school, and even in different sections in the same school teaching the same subject. There is truly little uniformity as to what is taught, how it is taught, and how understanding and proficiency is measured. ***This book is easy to read for learners of all ability levels, extremely thorough, and extremely ordered. Most math books are often useful for people who are already skilled in mathematics.***

This book is a great reference for teachers. It is my main resource for developmental and foundational math. It can be used as a stand-alone course as well as a resource for prerequisite skills needed when planning lessons for other subjects such as geometry, algebra, probability and statistics, financial math, physics, and other science courses. It has many more examples than most math books, and which cover most variations of any problem, and it offers practice on all those variations. It also explains why some methods are better than others in achieving results that will extend beyond the early years. This is always an area of contention for many teachers I talk with about why certain methods should be used over others. Just as teachers of early childhood learners know things about the developmental stage of children and how that translates to a classroom far better than I could because of their extensive training in that genre of education (yes, genre, as teaching is indeed an art), I have the perspective of a teacher of middle school and high school children and have only had to focus on one subject. Therefore, there are certain ways to do a problem in math that are better than others with respect to how it will translate to the later years. For instance, many students learn cross-cancelling when multiplying fractions. However, in my experience with older students, this many times leads to students confusing cross cancelling and cross multiplying which is why I believe students should learn how to cancel with prime factorization (which also eliminates the need to find a GCF to cancel). Another instance is finding factors. In mathematics at almost all grade levels, this is mostly learned by guessing and therefore students are given familiar numbers like 24 and 36. When I ask students in 9th grade is 91 is prime, the overwhelming majority says it is because they do not have a real method of finding factors. I find this particularly troublesome because students are asked to do and understand so much at a higher level that requires a solid understanding of factors. It is much more useful to learn how to use the primes and the square root method to find factors as early as possible (5th grade, 6th grade?). These skills open doors to rooms that were previously off limits to most students. As a guest teacher, I taught a 3rd grade class my "erase and replace" method of finding prime factorization without a factor tree (which can lead to losing factors, gaining factors, and missing the meaning of prime factorization). The students were excited and proud to learn this. ***Again, I have immense respect for teachers who deal with large groups of students with extreme ability level differences and all the complexities of the very young, as well having to know all subject areas (and many with a solid math background)***, as I know that other than raising my own children, that is not an area in which ***I*** am well versed. However, what I do know from my extensive experience in teaching older students mathematics is that there are methods that are more effective than others, and what I do see is the difficulties older students have in math by learning shortcut methods such as lattice multiplication that may by a good temporary fix but are many times dead ends on the road map of mathematics. ***As professionals we all need to check the ego at the door and work together, drawing from the strengths of each and every teacher at all levels. The ultimate goal has to be the success of the student.***

This book is a great reference and resource for administrators and public officials who design or otherwise effect the educational process. As I said in the first paragraph, we need to design a relevant, effective curriculum accessible to all. If students were to learn the skills in this book in the order they are presented and with the methods that are taught, they would be far ahead of where most students and adults are on the scale that measures the present state of mathematics proficiency in the United States. Too much time is spent on searching for the latest trend in teaching and evaluating which is evidenced by the fact that curriculum and methods of teaching and evaluation change all too frequently. Because of this, students and parents are never given the opportunity to grasp what it is they are learning and how they are learning it for very long. I have used this book as a guideline for tutoring and teaching students of various levels of ability and proficiency with enormous success. The fact that countless students have told me "why don't they teach it like that in school" speaks volumes.

How Is The Book Designed?
(Why is it different?)

This book is a reference, not a teacher. It is much more thorough and organized than any math book I have ever seen or used. I wrote it in a very specific order where I introduce concepts and build on them throughout the book. I wrote the book with the intention that people who have some background and understanding can teach themselves. Some books try to address all the new ways of teaching and learning such as cooperative learning, interdisciplinary learning, integrated subjects, real life connections, and activity-based learning. In addition, they are sometimes written to coincide with national or local standards and assessments whether or not they are effective and relevant. Some textbooks even go out of their way to make multicultural connections whether or not they are relevant to the subject matter. Although all these things have merit when not forced into a curriculum to satisfy some political agenda, they often become "clutter" that can cloud the subject matter being taught. There are all types of learners and all types of ways of learning. It is impossible to address all the different types of learning styles in this book. Although I believe experiential learning (learning by doing) is the way in which most of us learn, a book that tries to do that will become an experiment in chaos. That is the job of a teacher, not a book. The following list highlights some of the key elements of this book and some of the things that make this book unique when compared to other math books.

1. Math Explained Book I (Basic Math To Pre-Algebra) literally starts from the very beginning. It starts with the definition of a set and the set of digits from 0 – 9 that are used in the base 10 number system. It progresses through the set of whole numbers and operations on those sets. Operations with integers and negative numbers are covered in chapter 3. Chapter 4 covers operations on rational numbers (fractions) and decimals. Chapter 5 covers ratio, unit ratio, unit conversion by unit cancelling (which benefits students in science and math classes as well standardizing something that most students struggle with throughout math), percent and percent shortcuts, and an introduction of slope. Chapter 6 covers simplifying roots with both exact and approximate answers and the set of real numbers. Chapter 7 covers exponent rules including fractional roots, powers of 0, negative exponents, and scientific notation. Chapters 6 and 7 are usually taught in Algebra but should be taught as Pre-Algebra topics as students in high school quite often struggle with exponent rules and scientific notation. I included a brief chapter 8 covering number systems other than base 10, mod arithmetic, and Roman Numerals.

2. One of the best features of this book is the fact that it boasts many more examples than other math books. Some mathematicians feel that a few examples are sufficient for someone to be able to expand their knowledge to other variations of the exercises. However, in my experience this is generally not true. I feel that if there are 8 ways to write a problem covering a certain concept, then the book should demonstrate all 8 ways. In this way, students who need to see each variation will eventually gain a greater understanding, while students who gain understanding more quickly or have a more extensive math foundation, might not need to do every variation. All the books I have seen are written with too few examples or examples that are too complex for most students rather than progressing from the very basic to more complex problems. ***This allows all students "access to success***."

3. I wrote this book with an emphasis on vocabulary and the connection of the math definition to usage in everyday life. I did not do this to satisfy some agenda that says we need to do cross-curriculum teaching just to show we are doing it. I did it because the more you can associate the words with the concepts and processes in math by relating them to things you are more familiar with, the more you will gain in both your English and your math abilities. For instance, I introduce words very early on as needed to explain the material. Early in the book, I was teaching how to use an adding facts chart and realized I had to talk about rows and columns, and this made me realize I had to talk about horizontal and vertical, two terms students see a lot throughout mathematics and quite often confuse and misunderstand. I introduce conventions (unwritten rules) very early on in the book. I talk about the commutative property using the root word "commute" as in commute to work, and I talk about the associative property using the word "associate" as in associating with someone else to demonstrate the difference between the two properties as students confuse the two often. There are countless examples throughout the book of the rich vocabulary that can be learned and how it enhances mathematical understanding.

4. Repetition is an integral factor of the book as it is an integral factor of becoming proficient at anything we do in our lives. Although the ultimate goal is for students to be able to do the work accurately and understand what it is they are doing or what it means, sometimes we must do something over and over to gain both better proficiency and understanding. Repetition is a natural result of how this book is constructed. For instance, I introduce the definition of *definition*, and add definitions throughout as needed. I introduce number theory early on and add number theory as needed. Many concepts are built this way. Ways to represent numbers starts with standard form, then can be written in expanded form with place value, powers of 10, and finally negative powers of 10. Students learn how to find factors by dividing, then by divisibility rules, then by prime factors, and finally by the square root method. Properties are also added as needed and as we have enough knowledge to understand and use them. How to estimate math, shortcuts in math, and how to do the math on the TI-84 graphing calculator are all taught and improved upon throughout the book. In this way, the students have visited this concept many different times while progressing through the book, and this natural yet intentional repetition deepens understanding.

5. In the book I outline and explain many of the common errors, conventions, and misconceptions of mathematics.

6. I introduce the symbols of math as they are needed and used, and there is a complete list of the symbols used in the book in the appendices.

7. There are many appendices. These appendices cover topics such as common errors, definitions, the language of math, number theory, and formulas, rules and theorems to name a few.

8. There is a section on the algorithms used to simplify in math some of which are the steps to multiplying whole numbers and decimals, writing an improper fraction as a mixed number, and simplifying numerical expressions with real numbers using order of operations.

9. There are special reference sheets that have been immensely helpful to many of my students when I work with them. Some of them are all the sign rules from k – calculus, the steps to simplifying any exponent problem, the steps to simplifying any root problem, and everything you need to know about fractions and decimals.

10. Many times, certain concepts in math are not taught until Algebra. Such topics are covered in depth in this book. The following are some of the topics that should be taught as pre-algebra concepts and the advantages of doing this.

A. ***Pre-algebra should cover a thorough examination of the rules of roots and exponents***. Students generally are uncomfortable with roots and exponent rules by the time they start algebra. Students need to know what roots are, how to estimate roots, how to simplify roots, and the difference between an exact answer and an approximate answer. The advantages of this are that the students will better understand the rules for simplifying roots in algebra, that roots are the inverse operation of powers, how to put roots into a calculator as fractional powers, the Pythagorean Theorem and other geometry concepts involving roots such as 30-60-90 triangles, and where roots fit into order of operations. In addition, I find that students rarely know how to list the factors of a number by testing all the numbers up to the square root of the number. For the overwhelming majority of students, including students more advanced in math, finding factors is just a guessing game. There is also a thorough examination of exponent rules including negative and 0 powers as well as fractional exponents, their meaning and how to enter them into a TI-84 graphing calculator.

B. ***Pre-algebra should examine more thoroughly the concept of factors and other important number theory concepts.*** Number theory is largely left for just the "math" kids who make up a very small minority of students in any learning institution, yet it is so critical to understanding so many algebra concepts. Students should know how to find all the factor pairs of a number, how to prime factor a number, how to find how many factors there are using the prime factorization of a number, and the differences between listing factors, factor pairs, and prime factorization. Students should learn to reduce and other fraction concepts using prime factorization if we are to extend their learning to algebra rather than learning a lot of little tricks that do not transfer to algebra smoothly or at all. Other concepts that most students do not have a particularly good understanding of are why when the bottom of a fraction approaches 0 the fraction approaches infinity, when we square root a decimal less than 1 it gets larger, why we cannot divide by 0 but 0 divided by a number is 0, and many others. These are critical concepts in being able to estimate answers as well as understanding slope and other algebra concepts.

C. ***Pre-algebra should include a thorough explanation of ratios.*** There is much that can be done with this topic without the use of variables. Students need to understand the differences between a ratio and an actual quantity. Students who learn to turn ratios into equal unit ratios by dividing will understand slope better later (here is a good place to show a more concrete example of why division by zero yields an undefined number or infinity while dividing into zero yields 0). Students need to understand the difference between a rate and a ratio as it relates to units and whether it is appropriate to cancel units or why some have no units at all (unit-less ratios such as radians). These concepts become increasingly important as we climb through the progression of math courses. Students will understand percent better by seeing it as an equal ratio. Finally, there is an extensive section on unit conversion with unit cancelling which is a critical concept as it is an area of difficulty for students, and it appears in many problems in math and other subjects.

D. ***Pre-algebra should stress the importance of mathematical properties.*** Properties are usually taught as just another section in most math books. For Pete's sake, they are the backbone for why we can do anything. I believe they deserve just a little more respect than that, don't you? They are often forced into a section as an exercise unrelated to the many times (but not always) linear progression of mathematics. They should be taught when needed, repeated often, and used as justification for steps on problems leading to a better understanding of why something can or cannot be done as well as of deductive reasoning.

How Do You Get The Maximum Benefit From This Book?

Although someone who is proficient in math may need this book just as a reference or a reminder of how to do some math procedure, and a teacher may just need to look up a certain procedure and the prerequisite concepts that need to be taught to plan a lesson, for most it will be much more beneficial if the learner starts at the very beginning. ***Everything is important***. If you read a passage and it references a ***figure***, you should stop and look at the ***figure*** and make sure you understand what it means. If there is a ***definition***, make sure you read it and look at the ***example*** so that you fully understand the definition. Each example has an explanation of how to do the problem and why. You should read the examples and these explanations until you understand every step that was performed. In addition, one should not skip over the common errors. A lot of understanding can be gained from these. These common errors are mistakes I have seen many times throughout my years as a teacher. After reading an example, you should do the problems that go with that example before proceeding. ***Math is meant to be read slowly. Every word, picture, example, and figure matters. This book builds on each concept, and some concepts are developed further throughout the book.***

I encourage students to write out every step on every problem the way I model it in the book. Some students say, "but I do it differently." This is fine if it does not impede your understanding or progress. When I have students write things out a certain way, it is to help them "***see***" what it is that they are doing, find and correct mistakes more quickly, and ultimately gain a better understanding and proficiency with the topic. Ironically, students skip steps to save time, and in the long run it takes more time to do it again with less effective results. When doing the practice, it is not necessary to fully understand every concept before practicing. Although I try to explain concepts thoroughly and in plain terms, students may not fully understand the concept until they have practiced the problems using the examples and step-by-step instructions as a guide. Others may need the help of a teacher, tutor, or the lesson videos.

There are many reference sheets throughout the book that should be reviewed periodically and used when doing practice problems. Some of these are perfect squares, rules of operations with signed numbers, everything you need to know about fractions and sign rules, common errors, misconceptions, math habits and tips for quality thinking and work, number theory, conventions (unwritten rules) and common practice, definitions and symbols, shortcuts and mental math, the big picture, primes to 100, things you should memorize, theorems, and number sets.

Happy Learning!

Jeffrey Norman "Norris" Morris

Jeffrey Norman "Norris" Morris

Homework And Study Tips

Homework/Independent Practice Tips

When practicing for anything, we need to put in an "***honest***" effort. Sometimes students want to get the practice done as quickly as they can, so they look for shortcuts. If we want to be really good at playing the guitar or hitting a baseball or painting, we have to practice drills over and over. If we find ways of shortcutting the practice, we will never be able to perform those skills at a high level. The same is true for practicing mathematical skills. When practicing, we should follow the guidelines outlined below:

1. ***Always write the problem as it appears in the book***. Do not start doing the problem in your head so that by the time you start writing on your paper, the problem is already partly completed. This leads to computational errors as well as copy errors (copying something down wrong).

2. ***Use your resources*** such as the explanation in the book, the examples, the video lessons, the answers and solutions, your own notes, the reference cards provided in the book, and other students when doing work. Memorization is overrated. In real life, we look things up.

3. ***All problems should tell a story that is complete*** enough so that someone who reads your work can understand what the problem is, and how you arrived at the answer. The problem should have a beginning (the problem in the book), a middle (your complete work), and an end (an answer, many times in sentence form).

4. ***Do all the steps on all of the practice problems***, even if some of them are the same type of problem. Even when we understand something, we need to practice it 4 or 5 times before we fully understand it and so that we remember it later.

5. ***Do not look for patterns that will allow you to complete the homework faster***. Learning is not a race. Treat each problem individually by doing all the steps on *one problem at a time*. Do not try to work on several problems at the same time. Sometimes students try to get it done faster when the problems are the same type by doing step 1 on all problems, then step 2 on all problems, and so on. They are just trying to get it done fast and will probably not gain a very good understanding of the concept being practiced or remember it for very long. You get out of something what you put in for effort.

6. ***Do not avoid doing the problems when you already have the answers***. Sometimes students will tell me that multiple choice tests are easier because the answers are on the paper. Other times, I give students the answers so that they may check their work. Many times, I write out the solutions with every step. Students should do every problem as though they did not have the answers. Then, they should look to see if they got the correct answer or to help them if they get stuck. Looking at the answers before doing the problems will lead students to do incomplete work and lure them into choosing the wrong answer.

Note: There may be multiple methods to solve the same problem. However, whatever method works best for you, do not substitute expedience for understanding.

Tips For Studying And Mastery

1. ***<u>Read the material for the next class the night before the class, not after.</u>*** If you read the material the day before the class in which you will cover the material, you will already have some understanding and will therefore get a lot more out of class. You should highlight the areas you did not understand when you read the material. In this way you will know what questions you have ahead of time and ultimately get more out of class. This is especially critical in college where classes can be large, and a lot of material can be covered in one class setting.

2. ***<u>Complete all homework on time and exactly how you are instructed to do so.</u>*** Putting off homework will make you fall behind in class and eventually lesson your understanding of the material. Learning is easier in small "bites" with constant revisiting of the past material. Students who keep up with the assigned practice generally need to put in less time studying for a test or quiz and score higher than those who fall behind. ***Homework is for two primary purposes, to strengthen what you know and to find out what you do not understand***. Therefore, hurrying your homework just to "get it done" by not writing out your work will eventually lead to not understanding the material and doing poorly on evaluations. ***<u>Students should use the examples in their books and notes to ensure they are writing their work out properly.</u>***

3. ***<u>Putting in the time now will save you time later.</u>*** Reading the material ahead of class and writing out all your work will take more time in the beginning than not doing those things. However, putting in the extra time in the beginning of each section will lead to being able to do your homework faster later and also to having to study for a shorter period of time than those who shortcut right from the beginning. In addition, those who write out their work will find their mistakes much more easily.

4. ***<u>Practice what you are not good at.</u>*** Quite often students do all the problems they know how to do when they have review packets to prepare for evaluations. This may make them feel good, but it will lead to poor results. ***Students should highlight concepts and problems they are weak at or do not fully understand so they are ready to ask questions and they should practice those concepts***.

5. ***<u>Seek help from teachers and others.</u>*** Get help often when doing homework to make sure you understand the material. In addition, I suggest to many students that they may want to do their reviews with a teacher or parent present. Use your reference cards that are in this book. Reread the section on that material or watch the video lessons again. In addition to the video lessons that supplement this book, there are online tutors and other resources such as Khan Academy that provide explanations, video lessons, and practice free of charge or for a small fee.

6. ***<u>Write out and speak orally the steps to certain algorithms.</u>*** When studying for a math evaluation, it is a good practice to write out the steps to a certain math process several times on a piece of paper. In addition, students should be able to speak the steps orally from memory. Do this until you can do it without looking anything up. Students who do this are less likely to get "stuck" on a problem during a multi-step mathematical process. For instance, you might write and say the steps for simplifying with exponent rules: 1. Do powers to powers by multiplying exponents. 2. Multiply like bases by adding exponents. 3. Divide like bases by subtracting the bottom exponent from the top and leave the answer on top. 4. Move bases with negative powers and replace powers of 0 with 1. 5. Reduce with prime factorization.

CHAPTER TWO

Sets And Operations

2.1 Sets

•Definition •Set •Elements Of A Set • Commas And Lists •Digits 0 to 9 • Math Language And Symbols •Empty Set •Universal Set •Compliment Of A Set •Mutually Exclusive Sets •Intersection Of Sets •Union of Sets •Venn Diagram • Subset • Organized Systems

2.2 Set Of Whole Numbers

•Right and Left •Units And Magnitude •Units Place •Two-Digit Numbers •Regrouping •Exceptions To Naming Numbers •Three-Digit Numbers •Period •Writing And Naming Six-Digit Numbers •Place Value •Writing Numbers From Words •When Zeros Matter •Infinity, Infinite, Finite •Patterns •Different Ways To List Sets •Plotting/Reading Points On A Whole Number Line •Breaking A Graph •Coordinate •Natural Numbers •Whole Numbers •Distinct •Cardinal Numbers •Ordinal Numbers •Naming/Ordering Ordinal Numbers •Comparing Quantities •Theorem •Trichotomy Principal •Ordering Cardinal Numbers On A Number Line •Consecutive •Scale Of A Number Line •Ascending And Descending Order •Algorithm •Ordering Cardinal Numbers With Place Value •Estimate •Rounding (estimating) Whole Numbers •Truncating Whole Numbers •Practical Limitations And Considerations Of Rounding

2.3 Adding Whole Numbers

•Operation •Addition •Sum •Addends •Words For Addition •Writing Expression In Order Of The Words •Write Addition Expressions From Words •Numerical Expression •Simplify •Simplest Form •Vertical And Horizontal •Work Vertically (most of the time) •Add Without Regrouping •Add On A Calculator •To Memorize Or Not To Memorize •Add With Regrouping •Adding Facts Table 0 to 9 •Rows And Columns •Array/Matrix •Intersect •Adding Whole Numbers •Expanded And Standard Form Of Whole Numbers •Writing In Missing 0's •Property •Substitution Property •Commutative (Addition) •Associative Property (Addition) •Additive Identity •Compatible Numbers •Estimate Addition With Rounding •Convention •Adding With Mental Math •Introduction to Number Theory •Even And Odd Numbers •Number Theory Rules 1 – 4 (Add Even, Odd, Ones Place) •Deductive Reasoning •Inductive Reasoning •Conjecture •Proof

2.4 Subtracting Whole Numbers

•Subtraction •Difference •Minuend •Subtrahend •Words For Subtraction •Subtraction Is Not Commutative •Write Subtraction Expressions From Words •Subtract Without Regrouping On Number Line, Picture, Fingers •Subtract On A TI-84 Calculator •Inverse Operation •Subtraction

And Addition Inverse Operations •Subtract With Regrouping •Subtraction Facts Table 0 to 19 •Subtract Whole Numbers •Estimate Subtraction •Compatible Number And Mental Math With Subtraction

2.5 Multiplying Whole Numbers

•Multiplication •Product •Multiplicand •Multiplier •Factors •Symbols For Multiplication •Words For Multiplication •Multiplicative Property Of 0 •Multiplicative Identity (Property Of •Think "Operation" First •Commutative Property Of Multiplication •Associative Property Of Multiplication •Non-Negative Multiples •Specific Language •Skip Counting •Multiplication Facts Table (Memorize) •Compatible Numbers And Multiplication •Distributive Property Of Multiplication On Addition: Part I (One Term x More Than One Term) •Powers Of 10 (No Exponents) •Multiplying Powers Of 10 •Multiply One-Digit Number By Power Of 10 •Updated Expanded Notation (putting in 0's) •Multiply Any Whole Number By Power Of 10 • Multiply One-Digit Number By Number Ending In 0 •Multiply Whole Numbers Ending In All 0's Multiplying A One-Digit Whole Number By Any Whole Number •Distributive Property Of Multiplication: Part II(More Than One Term Times More Than One Term) •Multiply Whole Numbers •Multiplying Wholes On A Calculator •Multiply Whole Numbers That End In 0's •Estimate Multiplication With Rounding •Number Theory Rule 5 (Rounding Limits) •Number Theory 6 – 8 (Multiply Even/Odd)

2.6 Raising Whole Numbers To Powers

•Power •Exponent •Base •Base Factored Form •Simplifying Powers •Writing Out Your Work (Bus Driver) •1 to any power is 1 •Writing And Saying Powers •Squares And Cubes (memorize) •Powers Of 10 •Multiplying Powers Of 10 By Powers Of 10 •Multiplying Whole Numbers By A Power Of 10 •Expanded Notation With Powers Of 10 •Subsets And Exponents •Theorem 2: Number Of Subsets (Organized System) •Powers On A Calculator

2.7 Dividing Whole Numbers

•Division •Quotient •Dividend •Divisor •Symbols For Division •Words That Suggest Division •Writing Expressions For Division •Division Is Not Commutative •Name Parts Of Division •Divide With Pictures No Remainder •Division And Multiplication Inverses •Using Inverse Operations to Divide •Remainder •Long Division •Checking Division •Dividing Wholes with Long Division •Divide Larger Power Of 10 By Smaller Power of 10 •Divide Whole Numbers Ending In 0's •Math And Patterns •Finding the nth Element In A Pattern By Division •Even And Odd •Number Theory 9 – 11 (Dividing Even/Odd) •Factor •Multipliers Are Factors •Number Theory 12 (Divisor And Quotient Are Factors) •1 And The Number Are Factors Of All Whole Numbers •Number Theory 13 (Any Combination Of Multipliers Are Factors) •Factor Pair •List Factors With System •Finding Factors Using Long Division •Finding Factors Using The Multiplication Table •Number Theory 14 (Divisibility Rules) •Number Theory 15 (Factors Of Factors Are Factors, Multiples Of Non-Factors Are Not Factor) •Finding Factors Using Number Theory, Divisibility Rules, And Long Division •Prime And Composite •Number Theory 16 (Integers And Prime Factorization) •Prime Factorization •Compare Factors, Factor Pairs, And Prime Factorization •Finding Factors Using Prime Factorization (System) •Number Theory 17 (Number Of Factors) •Number Theory 18 (Sum Of Factors) •Using Prime Factorization To Get The Number Of Factors And The Sum Of The Factors •Greatest Common Factor (GCF) •Relatively Prime

•Finding GCF By Listing Factors •Number Theory 19 (GCF With Primes) •Finding GCF Using Prime Factorization •GCF Shortcuts •Least Common Multiple (LCM) •Finding LCM By Listing Multiples •Number Theory 20 (LCM With Primes) •Finding LCM By Using Prime Factorization •LCM Shortcuts •Long Division Method For GCF And LCM

2.8 Roots Of Whole Numbers

•Root •Radicand •Index •Roots And Powers Inverse Operations •Finding Roots Of Perfect Squares And Cubes •Any Root Of 1 is 1 •Finding Roots On A Calculator •Finding Roots With Trail And Error •Roots Of Even Powers Of 10

2.9 Simplifying Numerical Expression With Whole Numbers

•Equal Operations •Don't Do What Is Easy, Do What Is Correct •Associative Property Of Addition •Subtraction Is Not Associative •Subtraction Is Equal To Addition (Left To Right) •Associative Property Of Multiplication •Division Is Not Associative •Division Is Equal To Multiplication (Left To Right) •Factorial •Multiplication/Division Done Before Addition/Subtraction •Multiplication/Division "***Glue***" •Powers/Roots/Factorials Done Before Multiplication/Division •Grouping Symbols •Grouping Symbols Can Show Multiplication Or Change The Order Of Operations •Order Of Operations •Revisiting The Distributive Property •Star Operations •Grouping Symbols On A Calculator •Formula •Range •Formula For Calculating Scale Of A Graph •Steps For Calculating Scale Of A Graph •Performing Order Of Operations On Sets

•This book has a rich vocabulary that may extend well beyond math topics. These topics are usually reserved for much later on. The earlier you are introduced to them, the deeper you will understand them.

Definition 2.1.8 **Mutually Exclusive Sets**: Mutually exclusive sets are sets that have no common elements (elements that are in both set). In other words, if an element is in one of the sets, it can NOT be in the other one.

If Set A = {birds} and Set B = {dogs}, the sets are mutually exclusive. No bird is in the set of dogs, and no dog is in the set of birds.

Let us break down the words mutually exclusive separately. If we say something is mutual, such as the feeling is mutual, it means we both have the same feeling. If something is exclusive, you can only find it in one place. It excludes or leaves out all other places. Now if we put the two together, if something is mutually exclusive to all sets, it means excluding each other is mutual to all of them. If I am in set A, I exclude (cannot be in) set B. But since excluding is mutual to both sets, if I am in set B, I exclude (cannot be in) set A. They exclude each other. ***Bottom line, if sets are mutually exclusive, then no element can be in both sets***.

EXAMPLE 5 State if the sets are mutually exclusive or not mutually exclusive.

A. Set M = {all humans} Set F = {all girls}

Solution: Since a girl is also a human, girls can be in the set all humans and also the set all girls. Some elements are in both sets.

Answer: The sets are NOT mutually exclusive.

B. Set L = {left-handed people} Set R = {right-handed people}
(Assuming no ambidextrous people)

Solution: You are either right-handed (in set R) or left-handed (in set L), but you cannot be both (in both sets).

Answer: The sets are mutually exclusive.

•No stone is left unturned. I make no assumptions about what knowledge the reader has. Therefore, if I use the word left, I have to define it just as I must define the difference between horizontal and vertical or rows and columns. These and words like intersect are misunderstood quite often by students.

Left And Right

We will start this section with two definitions that will be needed in this section and that will be used frequently throughout math.

Definition 2.2.1 **Right (direction or side):** On the side of the body where the heart ***IS NOT*** mostly located or the direction EAST when a person is facing NORTH. See **Fig. 1**.

Fig. 1

Definition 2.2.2 **Left (direction or side)**: On the side of the body where the heart ***IS*** mostly located or the direction WEST(W) when a person is facing NORTH(N). See **Fig. 2**.

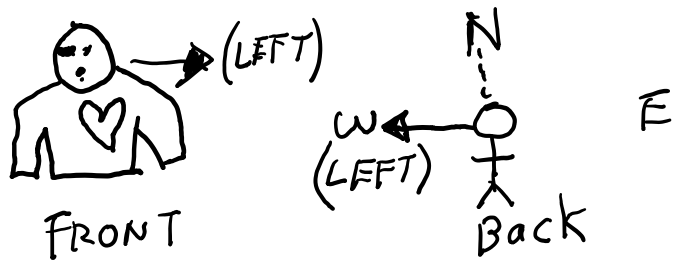


Fig. 2

- ***There are many concepts that are not emphasized enough or at all in most Pre-Algebra books. The sooner a learner is introduced to a concept, the more successful they will be with later concepts. I even included a Chapter on non-base 10 math such as Roman Numerals, Mod math, and different bases.***

Definition 2.9.1 **Factorial**: The factorial of a positive number is the product of all positive integers up to that number. The symbol for factorial is !. For example, 5! = 5 x 4 x 3 x 2 x 1 = 120. 0! = 1.

EXAMPLE 5 Simplify the expression 7!

$$\begin{aligned} & 7! \\ = \; & 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ = \; & 42 \times 20 \times 6 \\ = \; & 840 \times 6 \\ = \; & 5040 \end{aligned}$$

Since it is all multiplication, we can use the associative property and multiply more than once on the same line.

EXAMPLE 6 Simplify the expression $3! \bullet 6!$

$$\begin{aligned} & 3! \bullet 6! \\ = \; & (3 \bullet 2 \bullet 1) \bullet (6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1) \\ = \; & (6) \bullet (30 \bullet 12 \bullet 2) \\ = \; & 180 \bullet 24 \\ = \; & 4320 \end{aligned}$$

Ultimately, when we multiply two or more factorials, it is still all multiplication, so we can use the associative property and multiply any two numbers at any time.

Star Operations

There is another operation in math sometimes referred to as a star operation. Most operations are binary. That means they combine two numbers. Addition such as 5 + 7 is a binary operation because it combines the two numbers 5 and 7 into the one number 12. Sometimes we make up an operation based on a specific rule. Quite often we use the asterisk* (which looks like a star) as the operation symbol. This is why they are called star operations. Let us look at the star operation below.

The star operation rule is: $4*7=(4+7)^2$

This means whenever we put an asterisk between two numbers, we add the two numbers and square the sum (answer to addition). Therefore, using this rule:

$$8*3=(8+3)^2=11^2=121$$

- ***Repetition is a natural result of reading this book as it builds on concepts throughout. For instance, we learn order of operations with wholes. Then we add signed numbers. Next, we add fractions and decimals. Finally, we add irrational numbers and include approximate and exact answers.***

Order Of Operations With Real And Imaginary Numbers By Hand
(This is different if done with a calculator)

I. Set Up And Good Work Habits On All Problems

1. It is sometimes helpful to *underline* things connected by multiplication and division and from one end of parenthesis to the other. I call these "***clumps***." If there is a subtraction in front of parentheses and no number, write a 1 and replace all subtraction with "add the opposite" such as $^{-}3-(3-{}^{-}6)={}^{-}3+{}^{-}1(3+{}^{+}6)$. If there is a root, imaginary number, or irrational number like π or ϕ with no number in front, write in a 1. This helps us when we add like terms.

$$3-(2-12) \qquad 2-3\pi-\pi \qquad 2-3\phi-\phi \qquad 2-3\sqrt{7}-\sqrt{7} \qquad 2i-3+i$$
$$=3-1(2-12) \qquad =2-3\pi-1\pi \qquad =2-3-1\phi \qquad =2-3\sqrt{7}-1\sqrt{7} \qquad =2i-3+1i$$

2. Perform *ONE* operation at a time in each "***clump***" unless it is associative (all addition or all multiplication) using the sign rules. Think *operation, sign, magnitude, units.*

3. Work ***Vertically*** ($\updownarrow$), not *Horizontally* ($\leftrightarrow$). Bring down all parts (operations and numbers) that you have not used on each line.

4. If you are not sure of an answer in your head, do scratch work on the side of the problem (not in the middle of the problem) in a workspace.

5. When dealing with repeating decimals and irrational numbers, decide if the answers are approximate decimals or exact answers such as:

Exact

$$\frac{1}{3}+\frac{1}{3}+2e+3e+2\pi+5\pi+4\sqrt{3}-2\sqrt{3}$$
$$=\frac{2}{3}+5e+7\pi+2\sqrt{3}+2i$$

Approximate

$$\frac{1}{3}+\frac{1}{3}+2e+3e+2\pi+5\pi+4\sqrt{3}-2\sqrt{3}$$
$$\approx .333+.333+2\bullet 2.7+5\bullet 2.7+7\bullet 3.14+4\bullet 1.7-2\bullet 1.7$$

II. If There Are Fractions ....

- ***Number theory, a great critical thinking activity, is introduced early on and added to often. These are usually not emphasized until later. They can help us perform tasks much more efficiently and are critical in learning more advanced concepts. Many times, we learn similar concepts, but we do not take the time to understand the differences. Also, getting factors using primes or later square roots are two exact ways of finding factors that are taught very little or not at all. Another benefit is the thought process that goes into organizing things in a <u>systematic</u> way which can help in other areas.***

Before we move on in our quest for finding factors, now is a good time to distinguish between the terms ***<u>listing factors</u>***, ***<u>factor pairs</u>***, and ***<u>prime factorization</u>***. They are all similar, but they are used for different processes. Let us take the number 36 and examine the difference in the three terms.

<u>Factors</u>	**<u>Factor Pairs</u>**	**<u>Prime Factorization</u>**
Numbers that divide in with no remainder OR numbers that multiply to the number. We list them in order separating them with commas.	Pairs of numbers that multiply to the number. We list them pairing lowest to highest, next lowest to next highest, etc.	Writing the number as a product of prime factors. Order does not matter. We write them in base factored form or with exponents.
1, 2, 3, 4, 6, 9, 12, 18, 36	1 x 36 2 x 18 3 x 12 4 x 9 6 x 6	2 x 2 x 3 x 3 OR $2^2 \bullet 3^2$

EXAMPLE 37 List all the factors of 30 using the prime factorization.

Solution: Step 1 Prime factor the number (I used erase and replace).

30 = 5 x 2 x 3

Step 2 Use the prime factors to find all the factors.

1 factor
1 is a factor of all numbers
2 is a factor because 2 x (3 x 5) = 2 x 15 = 30
3 is a factor because 3 x (2 x 5) = 3 x 10 = 30
5 is a factor because 5 x (2 x 3) = 5 x 6 = 30

2 factors
5 x 2 = 10 is a factor because (5 x 2) x 3 = 10 x 3 = 30
5 x 3 = 15 is a factor because (5 x 3) x 2 = 15 x 2 = 30
2 x 3 = 6 is a factor because (2 x 3) x 5 = 6 x 5 = 30

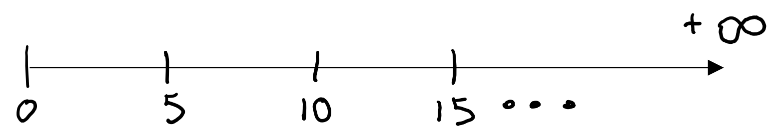
3 factors(all) 5 x 2 x 3 = 30 is a factor

Answer: The factors of 30 are **1, 2, 3, 5, 6, 10, 15, 30**.

- ***There are many common errors demonstrated throughout the book that I have seen in my over 30 years teaching many times. I point out the errors, the correct way, and explain the reasoning.***

COMMON ERROR 2 Graph the set $\{5, 10, 15, ...\}$.

Incorrect *Answer*:



Many times, students will write the numbers below the number line of the points they are supposed to graph. However, they will not color in a point on the graph. The lines and numbers just tell us the coordinates of the graphed points. The points are the actual graph, not the numbers. If we do not color in the points, it is like an empty measuring cup with lines and numbers on it. The lines tell us how much water we have in the cup. The ***water*** is the actual graph (like the ***points***). Just having numbers on the cup does not mean there is any water in the cup. Just having numbers on the number line does not mean there are any points on the graph.

- ***Math conventions (unwritten rules) are pointed out as well as it being another great vocabulary word. Convention #2 below is critical when we learn exponent rules.***

Convention #2 ***<u>If there is no exponent, then it is automatically assumed to be 1</u>***. For instance, the first power of 5 can be written as 5 or 5^1. In other words, if there is only 1 factor of the base, then we can choose to write the exponent or not. If the power is higher than that, we ***<u>must</u>*** write the exponent, or we would not know the power.

- ***Critical concepts that lead to success in higher levels of math are emphasized such as the importance of understanding why the unit is so important in determining the place of a number. Knowing what powers of 10 are and how to represent them is critical for success with division, multiplication, scientific notation, percent, many shortcuts, place value, and other areas.***

Unit And Magnitude

Definition 2.2.3 **Unit:** A single unit of any item is 1 of that item. How much you have of something depends on what you call ***one whole unit***.

Definition 2.2.4 **Magnitude:** The magnitude of something is how many units you have of that item. It is just ***"how many"*** you have. It is also called the ***amount*** or ***quantity***.

*****Cardinal Numbers have MAGNITUDE (how many) and UNITS (what type of thing each whole unit represents)**.***

Here are some of the powers of 10 in exponential and in standard form:

Exponential Form	Base Factored Form	Standard Form	Name
10 **or** 10^1	10	10	*ten*
10^2	10 x 10	100	*one hundred*
10^3	10 x 10 x 10	1,000	*one thousand*
10^4	10 x 10 x 10 x 10	10,000	*ten thousand*
10^5	10 x 10 x 10 x 10 x 10	100,000	*one hundred thousand*
10^6	10 x 10 x 10 x 10 x 10 x 10	1,000,000	*one million*
10^7	10 x 10 x 10 x 10 x 10 x 10 x 10	10,000,000	*ten million*
10^8	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10	100,000,000	*one hundred million*
10^9	10 x10 x10 x10 x10 x10 x10 x10 x10 x10	1,000,000,000	*one billion*

We can see the power is also the number of zeros. Therefore, when we multiply powers of 10, we just write a 1, and then we add up the exponents and this is the number of 0's we put after the 1. We are now ready to rewrite the power of 10 shortcut to include exponents.

Multiplying Powers Of 10 By Powers Of 10 (e.g. $10^3 \times 10^4$)

1. Write a 1.
2. Add the exponents. This is the number of 0's in the answer.

Example: $10^3 \times 10^4$ (3 + 4 = 7 zeros)

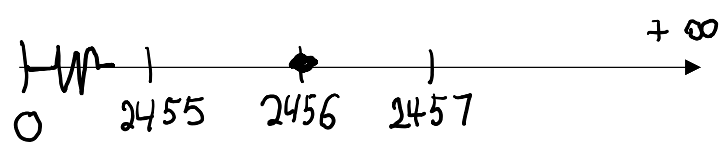
Answer: 10,000,000

Note: 300 is not a power of 10. It must begin with a 1 to use the property 1 x n = n.

- ***Graphing, ordering on a graph, breaking a graph, calculating the scale of a graph, and where to put 0 on a graph are all covered very early on as much of math beyond Pre-Algebra involves graphing and interpreting graphs. Once, again, the earlier we are introduced to these concepts, the more successful and confident we will be.***

Sometimes when we graph numbers that are not close to 0, we must ***"break the graph"*** so we can show the number we are graphing ***and*** the origin (starting point) 0.

EXAMPLE 18 Graph the number 2456. Use a broken graph to show the origin 0.



Now that we know order of operations, we can learn how to calculate the scale using the following steps below.

Steps For Calculating The Scale Of A Graph

1. Use the scale formula:

$$\textbf{Scale} \approx \frac{\textbf{Range}}{\textbf{Number Of Marks}} \approx \frac{\textbf{Largest Number} - \textbf{Smallest Number}}{\textbf{10 to 20}}$$

2. Round to a convenient number like 2 or 5 or 10. If we round up, we will get fewer marks. If we round down, we will get more marks.

- ***There are many examples covering all variations of a problem rather than leave it up to the reader to figure it out from a few examples.***

EXAMPLE 1 Simplify the expression ${}^{-}3+5^2-(2-10)^2$

$${}^{-}3+5^2-(2-10)^2$$

$$= {}^{-}3+5^2+{}^{-}1(2+{}^{-}10)^2$$

$$= {}^{-}3+25+{}^{-}1({}^{-}8)^2$$

$$= {}^{-}3+25+{}^{-}1({}^{+}64)$$

$$= {}^{-}3+25+{}^{-}64$$

$$= {}^{+}22+{}^{-}64$$

$$= {}^{-}42$$

1. Put a 1 in front of the parentheses. Replace all subtraction with add the opposite. Underline clumps (connected by multiplication, division and parentheses).
2. In each clump do one operation at a time. Bring down the -3. Square the 5. Do inside the parentheses.
3. Bring down the -3 and 25. Square the -8. (-8 x -8 = +64)
4. Bring down the -3 and 25. Multiply the -1 x 64.
5. Add two at a time.

EXAMPLE 3 Simplify the expression ${}^{-}3\bullet\left(\frac{2}{3}\right)^2-\frac{1}{2}\left(2\frac{1}{2}-4\right)\div\frac{2}{3}$.

$${}^{-}3\bullet\left(\frac{2}{3}\right)^2-\frac{1}{2}\left(2\frac{1}{2}-4\right)\div\frac{2}{3}$$

$$=\frac{{}^{-}3}{1}\bullet\left(\frac{2}{3}\right)^2+{}^{-}\frac{1}{2}\left(\frac{5}{2}+{}^{-}\frac{4}{1}\right)\bullet\frac{3}{2}$$

$$=\frac{{}^{-}3}{1}\bullet\left(\frac{4}{9}\right)+{}^{-}\frac{1}{2}\left({}^{-}\frac{3}{2}\right)\bullet\frac{3}{2}$$

$$=\frac{{}^{-}3}{1}\bullet\left(\frac{4}{9}\right)+{}^{-}\frac{1}{2}\left({}^{-}\frac{3}{2}\right)\bullet\frac{3}{2}$$

$$=\frac{{}^{-}4}{3}+{}^{+}\frac{9}{8}$$

$$=\frac{{}^{-}5}{8}$$

1. Write mixed numbers as improper fractions. Write whole numbers over 1. Rewrite division as multiplying by the reciprocal. Replace subtraction with add the opposite. Underline clumps.
2. In the first clump we must do the power first ($\frac{2}{3}\bullet\frac{2}{3}=\frac{4}{9}$), and in the second clump do inside parentheses first ($\frac{5}{2}+{}^{-}\frac{4}{1}\bullet\frac{2}{2}=\frac{5}{2}+{}^{-}\frac{8}{2}={}^{-}\frac{3}{2}$).
3. In both clumps, multiply (${}^{-}3\bullet\left(\frac{2}{3}\right)^2-\frac{3\bullet4}{9}=-\frac{\cancel{3}\bullet2\bullet2}{\cancel{3}\bullet3}=-\frac{4}{3}$ and ${}^{-}\frac{1}{2}\left({}^{-}\frac{3}{2}\right)\bullet\frac{3}{2}=-\frac{9}{8}$. Use sign rules for multiplication.
4. Add fractions. Get common denominators, equal fractions, and add the tops using adding sign rules $\frac{{}^{-}4}{3}\bullet\frac{8}{8}+{}^{+}\frac{9}{8}\bullet\frac{3}{3}=\frac{{}^{-}32}{24}+\frac{27}{24}=\frac{{}^{-}5}{24}$.

- ***Topics that will help in Algebra such as inverse operations, when to get an exact answer versus approximate answer, and fractional powers (which most students do not learn in any depth until precalculus) are covered thoroughly.***

<u>**A Root is the inverse operation of a corresponding power. A square root is the inverse of the second power, a third root is the inverse of the third power, and so on.**</u>

EXAMPLE 3 Simplify the expression 4^3. Then use the inverse operation to check.

	<u>Start With</u>	**<u>Raise To The *Third* Power</u>**		**<u>Answer (Power)</u>**
Simplify:	4	4^3	=	64

Check: If we raise 4 to the *third* power, the answer is 64. The reverse of the *third* power is taking the *third* root of the answer.

<u>Start With Answer</u> (what we ended with)	**<u>Take The *Third* Root</u>** (inverse of *third* power)		**<u>Answer (Root)</u>** (What we started with)
64	$\sqrt[3]{64} = \sqrt[3]{4^3}$	=	4

EXAMPLE 6 Simplify the following.

A. Simplify and write the exact answer of $2\pi + 3\pi + 5$

Solution: Since we want the exact answer, we must leave the symbol π in the answer.

Answer: $5\pi + 5$

B. Simplify $\sqrt{72}$

Solution: Since we did not say approximate, convention #7 tells us to assume we need to find the exact answer.

$$\sqrt{\cancel{2 \bullet 2} \bullet 2 \bullet \cancel{3 \bullet 3}}$$
$$= 2 \bullet 3\sqrt{2}$$
$$= 6\sqrt{2}$$

Answer: $6\sqrt{2}$

C. Approximate the value $3\phi - 4$ to the thousandths place. Use $\phi \approx 1.618$.

Solution: Since we said to approximate, we put in an approximation for phi. As soon as we approximate a number, we replace the equal sign (=) with the approximately equal sign $\approx$.

$$\begin{aligned} &3\phi - 4 \\ &\approx 3(1.618) - 4 \\ &\approx 4.854 - 4 \\ &\approx .854 \end{aligned}$$

Answer: .854

Fractional Exponents

(A fractional exponent is both a root and a power.)

$9^{\frac{1}{3}} = \sqrt[3]{9}$ $\quad$ $9^{\frac{1}{n}} = \sqrt[n]{9}$

$8^{\frac{2}{5}} = \left(\sqrt[5]{8}\right)^2 = \sqrt[5]{8^2}$ $\quad$ $x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a = \sqrt[b]{x^a}$

EXAMPLE 2 Write the following in radical form.

A. $15^{\frac{1}{5}}$ B. $7^{\frac{5}{3}}$

Solution: A. $15^{\frac{1}{5}} = \sqrt[5]{15}$

B. $7^{\frac{5}{3}} = \left(\sqrt[3]{7}\right)^5$ or $\sqrt[3]{7^5}$

There too many examples to list of what sets this book apart. In addition to teaching the basics in a very complete and organized way, it sets the foundation for advanced topics found in algebra, precalculus, and calculus such as set builder and interval notation, limits of infinity, significant figures, accuracy verse precision, fractional exponents, rules of roots and exponents, sum notation, sequences, continuous sets verse discrete sets, conjugates, imaginary numbers, and hierarchy of operations (which ones grow or get big the fastest - very important in graphing functions as well as critical thinking). It even includes an example of how to find the square root of a number longhand, and although probably not necessary, but I thought it should be in the book:)