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Fractions

And

Decimals

Improper Fractions And Mixed Fractions

Converting An Improper Fraction To A Mixed Number

$$\frac{Numerator}{Denominator} \rightarrow Denominator\overline{\big)\,Numerator}^{\;Wholes}\;\; Remainder \rightarrow Wholes\frac{Remainder}{Denominator}$$

Example: Improper Fraction $\frac{14}{5}$ Divide $5\overline{\big)\,14}^{\;2}$, $-\,10$, 4 $\quad 2\frac{4}{5}$

Converting A Mixed Number To An Improper Fraction

$$Wholes\frac{Numerator}{Denominator} = \frac{Denominator \times Wholes + Numerator}{Denominator}$$

Example: $2\frac{4}{5} = \frac{5 \times 2 + 4}{5} = \frac{14}{5}$

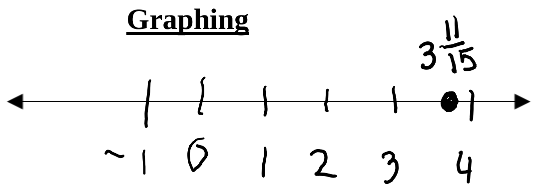
Whether To Use A Mixed Number Or An Improper Fraction

When we are *simplifying* fractions with order of operations, it is most often better to have it as an *improper* fraction. When we ***graph*** an improper fraction, it is better to graph it as a mixed number. ***<u>We do not always have to convert an improper fraction to a mixed number. However, we should always reduce.</u>***

<u>Simplifying</u>

$$2\frac{1}{3} + 1\frac{2}{5} = \frac{7}{3} + \frac{7}{5} = \frac{35}{15} + \frac{21}{15} = \frac{56}{15} \text{ OR } 3\frac{11}{15}$$

<u>Graphing</u>



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Step-By-Step

Algorithms

Simplifying Roots

Steps For Simplifying Roots

1. Break the number inside the root into its prime factorization.

$$3\sqrt{150} = 3\sqrt{2 \bullet 3 \bullet 5 \bullet 5}$$

2. For every ***two*** of the same factor you cross off on the ***inside*** of the root, put ***one*** of that factor on the ***outside*** of the root.

$$3\sqrt{150} = 3 \bullet 5\sqrt{2 \bullet 3 \bullet \left(\cancel{5 \bullet 5}\right)}$$

3. Multiply together all numbers on the outside of the root. Multiply together all numbers on the inside of the root.

$$3 \bullet 5\sqrt{2 \bullet 3 \bullet \left(\cancel{5 \bullet 5}\right)} = 15\sqrt{6}$$

<u>*****NEVER MULTIPLY A NUMBER ON THE OUTSIDE OF THE ROOT WITH A NUMBER ON THE INSIDE OF THE ROOT*****</u>

Steps For Multiplying Roots

1. Put all numbers outside together and put all numbers inside the roots under one root.

$$3\sqrt{20} \bullet 2\sqrt{10} = 3 \bullet 2\sqrt{20 \bullet 10}$$

2. Simplify the root.

$$3 \bullet 2\sqrt{20 \bullet 10} = 3 \bullet 2\sqrt{\left(\cancel{2 \bullet 2}\right) \bullet \left(\cancel{5 \bullet 5}\right) \bullet 2} = 3 \bullet 2 \bullet 2 \bullet 5\sqrt{2}$$

3. Multiply any numbers outside the root together and multiply any numbers inside the root together. Never multiply numbers outside the root with numbers inside the root.

$$3 \bullet 2 \bullet 2 \bullet 5\sqrt{2} = 60\sqrt{2}$$

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Rules

Of

Signed

Numbers

Negatives and Exponents

1. Negative Base To An Even Power Is Always Positive

$$(^{-}5)^2$$
$$= (^{-}5)(^{-}5)$$
$$= {}^{+}25$$

2. Negative Base To An Odd Power Is Always Negative

$$(^{-}5)^3$$
$$= (^{-}5)(^{-}5)(^{-}5)$$
$$= {}^{-}125$$

3. Positive Base Multiplied By A Negative Is Always Negative Whether The Exponent Is Even Or Odd

$$^{-}5^2 \qquad\qquad {}^{-}5^3$$
$$= {}^{-}1(^{+}5)(^{+}5) \quad \text{OR} \quad = {}^{-}1(^{+}5)(^{+}5)(^{+}5)$$
$$= {}^{-}25 \qquad\qquad = {}^{-}125$$

Negative Exponents:

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25} \quad \text{and} \quad \frac{1}{5^{-2}} = \frac{5^2}{1} = 25$$

Roots and Negatives

1. Even roots of negative numbers do not exist in set of real numbers (they are imaginary).

$$\sqrt{^{-}4}$$
$$= \text{No Real Answer}$$

$(^{+}2)(^{+}2) \neq {}^{-}4$
AND
$(^{-}2)(^{-}2) \neq {}^{-}4$

2. Odd roots of negative numbers are negative

$$\sqrt[3]{^{-}8}$$
$$= \sqrt[3]{(^{-}2)(^{-}2)(^{-}2)}$$
$$= {}^{-}2$$

CHECK
$(^{-}2)(^{-}2)(^{-}2)$
$= {}^{-}8$

Imaginary Numbers

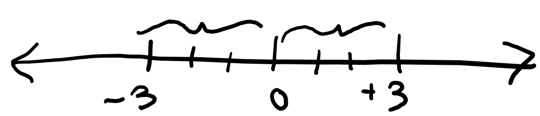
$$i^2 = {}^{-}1 \rightarrow \sqrt{^{-}1} = \sqrt{i^2} = i \quad \text{and so } \sqrt{^{-}4} = \sqrt{^{-}1(^{+}4)} = \sqrt{i^2 \bullet 4} = 2i$$

$$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = +1 \quad \text{(pattern repeats)}$$

Absolute Value:

Magnitude or distance from zero of a number. Always positive.

$$|^{-}3| = {}^{+}3 \quad \text{and} \quad |^{+}3| = {}^{+}3$$



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Unit Conversion
With
Unit Cancelling
And
Unit Ratio

The Four Basic Unit Conversion Types

There are four basic conversion situations. They are as follow:

1. ***<u>Converting in the same system (English or metric) and same type (length, area, space volume, liquid volume, time, weight)</u>***. For instance, we may have English system units of type length, and want to convert to a different English system unit of type length such as miles (mi.) to feet (ft.).

$$25 \text{ mi.} \times \frac{5280 \text{ ft.}}{1 \text{ mi.}} = 132,000 \text{ ft.}$$

2. ***<u>Converting between systems in the same type</u>***. For instance, we may want to convert from metric system units of type volume to an English system unit of type volume such as centimeters (cm) to feet (ft.).

$$200 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \approx 6.562 \text{ ft.}$$

3. ***<u>Converting between different types</u>***. For instance, we may want to convert liquid volume type to solid volume type such as gallons (gal.) to cubic feet (cu. ft.). We may want to convert liquid volume gallons (gal.) to weight pounds (lb.).

$$22 \text{ gal} \times \frac{8.34 \text{ lb.}}{1 \text{ gal}} = 183.48 \text{ lb.}$$

4. ***<u>Compound units</u>***. For instance, we may have miles per hour and want to convert to feet per second. Since miles (mi.) and feet (ft.) are type ***length***, and hours (h) and seconds (s) are type ***time***, we must convert one type at a time.

$$\frac{25 \text{ mi.}}{1 \text{ hr}} \times \frac{5280 \text{ ft.}}{1 \text{ mi.}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{132,000 \text{ ft.}}{3600 \text{ s}} = 36.\bar{6} \ \frac{\text{ft.}}{\text{s}}$$

Length (distance) Conversion Chart

English And U.S. Customary Units	***Bridge Units***	***Metric System Units***
12 in. = 1 ft.	1 in. = 2.54 cm	1,000,000,000,000 pm = 1 m
3 ft. = 1 yd.	1 mi. = 1.6093 km	1,000,000,000 nm = 1m
220 yd. = 1 fur	.6214 mi. = 1 km	1,000,000 μm = 1 m
16.5 ft. = 1 rod		1,000 mm = 1 m
6076 ft. = 1 nmi or NM		100 cm = 1 m
1760 yd. = 1 mi.		10 dm = 1 m
5,878,623,400,000 mi. = 1 ly		10 m = 1 dam
92,955,807.3 mi. = 1 AU		100 m = 1 hm
3.27 ly = 1 pc		1000 m = 1 km
		1,000,000 m = 1 Mm
		1,000,000,000 m = 1 Gm
		9,460,700,000,000,000 m = 1 ly
		149,593,781 km = 1 AU

Abbreviations

in. = inch	pm = picometer
ft. = foot	nm = nanometer
yd. = yard	μm = micrometer
mi. = mile	mm = millimeter
ly = lightyear	cm = centimeter
AU = astronomical unit	dm = decimeter
fur = furlong	m = meter
nmi or NM = nautical mile	da = deka/decameter
pc = parsec	hm = hectometer
	km = kilometer
	Mm = megameter
	Gm = gigameter

In general, the most closely related units of length in the English and metric systems are:

1/32 inch $\leftrightarrow$ millimeter (very small distances like paper width, bug wing length)
inch $\leftrightarrow$ centimeter (small distances like height of person)
yard $\leftrightarrow$ meter (house and land measurements)
mile $\leftrightarrow$ kilometer (large distances on earth and space)
ly and AU – both systems (huge distances in space, the universe)

Note: There are many other conversions you will see, like 36 in. = 1 yd. or 5280 ft. = 1 mi. or .6214 mi. = 1 km and many more. However, if you use unit cancelling it does not matter which one you use. The units will force the numbers to be in the correct place (top or the bottom of the fraction).

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Addition Facts Table

Multiplication Facts Table

Perfect Squares To 900

Perfect Cubes To 1000

Perfect Square Roots To 900

Perfect Cube Roots To 1000

Prime Numbers To 100